

KUNS-1436
HE(TH) 97/03
hep-th/9703077

A Comment on Fundamental Strings in M(atrix) Theory

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Abstract

We present a solution of M(atrix) theory describing type IIA fundamental string. Our construction is based on the central charge of the longitudinal membrane (= fundamental string), the BPS saturation condition and the relation between M(atrix) theory and supersymmetric Yang-Mills theory. The fundamental string corresponds to a photon in supersymmetric Yang-Mills theory.

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[†]Supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science and Culture (#5416).

1 Introduction

Recently Banks, Fischler, Shenker and Susskind [1] proposed M(atrix) model as a nonperturbative unified description of M-theory [2] and all the string theories. They claimed that the large N limit of a supersymmetric matrix quantum mechanics describing N D-particles is equivalent to M-theory. To confirm this conjecture, many nontrivial consistency checks were carried out to attain desirable results. For example, it was shown [1] that the D2-brane tension and the long range force between moving D-particles coincide with the values expected from supergravity arguments. Similar checks were also done in [3, 4]. Other even dimensional objects, such as D4 and D6-brane, were also constructed, and the potentials between them were calculated to agree with string theory expectations[5].

It is believed that in eleven dimensional supergravity there are two dimensional object (M2-brane) and five dimensional one (M5-brane), which are coupled to the three-form field $A_{\alpha\beta\gamma}$ and its dual field $\tilde{A}_{\mu_1\ldots\mu_6}$, respectively. These objects correspond to D-objects, fundamental string and NS5-brane in type IIA string theory. The consistency of the correspondence was analyzed in detail in [6]. Among these objects, D0, 2, 4-branes have been constructed in M(atrix) theory [1, 7]. If M(atrix) theory describes all the states of M-theory as expected, fundamental string (longitudinal membrane) and NS5-brane (transverse M5-brane) must also be constructed in M(atrix) formulation.

As for transverse M5-brane, Banks et al. [7] analyzed the supersymmetry algebra in M(atrix) theory and found that the central charge of transverse M5-brane is missing from the algebra. This might imply the impossibility of constructing transverse M5-brane in M(atrix) theory. On the other hand, longitudinal M2-brane charge exists in the M(atrix) theory supersymmetry algebra [7] and it is constructed from variables X_a and θ . This fact strongly suggests the existence of longitudinal M2-brane (fundamental string) solution of M(atrix) theory.

The purpose of this paper is to construct the solution describing the fundamental string in M(atrix) theory. To get the solution, we study the expression of the string central charge in M(atrix) theory supersymmetry algebra[7] and use the BPS saturation condition. It is found that the fundamental string corresponds to the “photon” in supersymmetric Yang-Mills theory (SYM).

We shall comment on earlier works [1, 8, 9, 10] constructing fundamental string solution in M(atrix) theory. In [1], it was claimed that if we compactify one of the transverse directions and T-dualize the theory along that direction, a string world sheet action is obtained, which they identified as that of type IIA fundamental string. Because the one dimensional object

which is obtained by T-dualization of M(atrrix) theory is not a fundamental string but a D-string, we need further S-dualization to identify it with the fundamental string¹. The fundamental string solution we present in this paper is the one in non-S-dualized description, and therefore is different from the fundamental string discussed in [1, 8, 9, 10]. Indeed, the same objects are described in quite different ways in each formulation. For example, in the non-S-dualized description, D-particles are represented by diagonal elements of matrices, while from the viewpoint of S-dualized description it is represented as an electric flux on the world sheet.

We shall add a comment on the longitudinal radius R . If we want to reproduce (uncompactified) eleven dimensional theory, we must take the limit $R \rightarrow \infty$. However, because our purpose is to study fundamental string, we should keep the string coupling constant weak, implying small R . Therefore in this paper we use the words “M-theory” and “M(atrrix) theory” as eleven dimensional theory compactified on a small S^1 , although they usually imply a decompactified theory.

Finally, we present a few formulas used in later sections. In the M-theory context, ten dimensional type IIA theory with coupling g_{str} is equivalent to eleven dimensional supergravity compactified on $S^1(R)$ with radius R related to the string coupling by

$$R = g_{\text{str}}^{\frac{2}{3}} l_s, \quad (1)$$

where l_s is the string length scale (we shall adopt the convention $l_s = 1$ hereafter). Another important relation is the rescaling formula of metrics,

$$G_{\mu\nu}^{(11)} = g_{\text{str}}^{-\frac{2}{3}} G_{\mu\nu}^{(10)}. \quad (2)$$

This equation is usually expressed as $l_{\text{planck}} = g_{\text{str}}^{-\frac{1}{3}} l_s$.

2 Central charges of branes

Ten dimensional SYM theory has 16 supersymmetry transformation parameters ϵ . In addition to them, 16 trivial fermionic symmetries $\delta\theta = \tilde{\epsilon}$ exist. The idea of M(atrrix) theory is to regard these 32 fermionic symmetries

$$\delta X_a = \frac{i}{2} \epsilon \gamma_a \theta, \quad \delta\theta = -\frac{1}{2} D_t X_a \gamma^a \epsilon - \frac{1}{8\pi} [X_a, X_b] \gamma^{ab} \epsilon + \tilde{\epsilon}, \quad (3)$$

¹In the theory thus obtained by T- plus S-dualizations of M(atrrix) theory, all kinds of extended objects are described in terms of two dimensional S_N orbifold sigma-model [8, 9, 10]. This S-dualization prescription is equivalent to the exchange of two directions x^9 and x^{11} as is explained in [10].

as a superspace representation of the eleven dimensional supersymmetry transformations [1]. We can construct supercharges for these transformations as follows [7].

$$\epsilon Q = \frac{\sqrt{R}}{2} \text{Tr} \left(P_a (\epsilon \gamma^a \theta) \right) + \frac{\sqrt{R}}{8\pi} \text{Tr} \left([X_a, X_b] (\epsilon \gamma^{ab} \theta) \right), \quad (4)$$

$$\tilde{\epsilon} \tilde{Q} = \frac{1}{\sqrt{R}} \text{Tr} (\tilde{\epsilon} \theta). \quad (5)$$

If we omit the second term on the RHS of eq. (4), these supercharges reproduce eleven dimensional ordinary (= no central charge) supersymmetry algebra. The omitted term can be regarded as contributions from objects carrying the central charges.

Banks et al.[7] obtained the expressions of central charges of extended objects except the one for transverse N5-brane by calculating the anti-commutation relations of supercharges. Their result is

$$Z_b = -i \frac{R}{2\pi} \text{Tr} \left(P_a [X_a, X_b] \right) - i \frac{R}{4\pi} \text{Tr} \left(\theta^\alpha [\theta^\alpha, X_b] \right), \quad (6)$$

$$Z_{ab} = \frac{i}{4\pi} \text{Tr} [X_a, X_b], \quad (7)$$

$$Z_{abcd} = \frac{R}{8\pi^2} \text{Tr} X_{[a} X_b X_c X_{d]}. \quad (8)$$

These are the central charges of longitudinal membrane, transverse membrane and longitudinal 5-brane, respectively. Their R -dependence is consistent with the fact that the corresponding brane is longitudinal or transverse. To be precise, we have to compactify the transverse directions in order for these central charges to be well-defined.

Let us pay attention to the one brane charge Z_a related to longitudinal M2-brane. In the type IIA picture, it is the wrapping number of fundamental string. If a BPS saturated solution with nonzero Z_a exists, its energy is equal to $|Z_a|$. Thus the following equation holds (neglecting the fermionic part):

$$H = \frac{R}{2\pi} \left| \text{Tr} (P_a [X_a, X_b]) \right| = \frac{1}{2\pi} \left| \text{Tr} (\dot{X}_a [X_a, X_b]) \right|, \quad (9)$$

where we have adopted the eleven dimensional supergravity metric. Translating eq. (9) to the string metric by using eq.(2) and rescaling the variables as $X_a \rightarrow g_{\text{str}}^{-1/3} X_a$, $t \rightarrow g_{\text{str}}^{-1/3} t$, $H \rightarrow g_{\text{str}}^{1/3} H$, we obtain

$$H = \frac{1}{2\pi g_{\text{str}}} \left| \text{Tr} (\dot{X}_a [X_a, X_b]) \right|. \quad (10)$$

3 Relation to Supersymmetric Yang-Mills theory

In [11, 12] it was shown that M(atrix) theory compactified on a torus T^p with radius R_a ($1 \leq a \leq p$) is equivalent to $p + 1$ dimensional SYM theory on the dual torus \tilde{T}^p with radius $\tilde{R}_a = 1/R_a$ via T-duality transformation. A D-particle in IIA theory is translated to background p -brane wrapped around \tilde{T}^p , and the variable X_a to $U(N)$ gauge field A_a .

Compactification means that X_a and $X_a + 2\pi R_a$ are the same point, and this fact is translated to that A_a and $A_a + 1/\tilde{R}_a$ are gauge equivalent by a large gauge transformation $\exp(ix_a/\tilde{R}_a)$. Thus a relation between X_a and A_a is

$$X_a = 2\pi \nabla_a \equiv 2\pi (\partial_a + A_a). \quad (11)$$

Therefore, the membrane wrapping number $[X_a, X_b]$ and the velocity \dot{X}_a in M(atrix) theory correspond to the magnetic flux $(2\pi)^2 B_{ab} = [2\pi \nabla_a, 2\pi \nabla_b]$ and the electric flux $2\pi E_a = 2\pi \dot{A}_a$ in SYM, respectively. The relation between the coupling constants g_{str} and g_{YM} can be derived by comparing the two actions in the $10 - p$ uncompactified dimensions:

$$\frac{(2\pi)^2}{g_{\text{str}}} = \frac{V}{g_{\text{YM}}^2}, \quad (12)$$

where V is the volume of the dual torus \tilde{T}^p .

4 Fundamental string solution

Since the following argument does not depend essentially on p , we shall consider the most familiar case $p = 3$. In this case, the magnetic flux is represented by a 3-vector $B_a = \frac{1}{2}\epsilon_{abc}B_{bc}$. Rewriting eq. (10) by using the correspondence (11), we have

$$H = \frac{1}{g_{\text{YM}}^2} \left| \int dV \text{Tr} (E \times B) \right|. \quad (13)$$

In deriving this equation, we have used the expression of the central charge (6) and the BPS saturation condition $H = |Z_a|$.

On the other hand, if we regard a configuration carrying nonzero Z_a as a string, its energy is given as a product of string tension $1/2\pi$ and the circumference $2\pi R_3$,

$$H = R_3 = \frac{1}{R_3}. \quad (14)$$

(We assume that the winding is along the 3rd direction, which implies that $Z_3 \neq 0$. In the M-theory view point, we are considering the M2-brane extended in the longitudinal and 3rd directions.) Combining eqs. (13) and (14), we obtain the key equation which the fundamental string configuration should satisfy:

$$\frac{1}{\tilde{R}_3} = \frac{1}{g_{\text{YM}}^2} \left| \int dV \text{Tr} (E \times B) \right|. \quad (15)$$

This equation is interpreted as follows. The LHS is one unit of Kaluza-Klein momentum and the RHS is the Poynting vector of Yang-Mills field. Therefore, eq. (15) tells us that the configuration corresponding to fundamental string wrapped around the 3rd direction is a plane wave of one photon moving along the 3rd direction. To be precise, unless the number of photons is sufficiently large, it is impossible to determine the field strength (E and B) and the photon momentum simultaneously owing to the uncertainty principle. Here we shall ignore this problem and consider a classical plane wave with Poynting vector $1/\tilde{R}_3$.

First let us consider the plane wave taking values in the $U(1)$ part of $U(N)$. The field strength of such a plane wave is

$$E_1 = B_2 = \frac{g}{2\pi^{3/2}\sqrt{\tilde{R}_1\tilde{R}_2\tilde{R}_3}} \sin \frac{x_3 - t}{\tilde{R}_3} H_i, \quad \text{others} = 0, \quad (16)$$

where $(H_i)_{mn} = \delta_{im}\delta_{jn}$ ($i = 1, \dots, N$) are the generators of the $U(1)$ part. The corresponding vector potential is given by

$$A_1 = c \cos \frac{x_3 - t}{\tilde{R}_3} H_i, \quad A_2 = 0, \quad A_3 = \sum_k \frac{a_k}{\tilde{R}_3} H_k. \quad c = \frac{g}{2\pi^{3/2}\sqrt{\tilde{R}_1\tilde{R}_2}}, \quad (17)$$

where a_k are Wilson lines along the 3rd direction. For simplicity, we have assumed that A_1 and A_2 have no static parts, implying that all the D-particles sit on the x^3 axis. This result looks natural if we note that type IIA string theory and SYM theory are T-dual with one another, and under this duality wrapped string in type IIA theory is transformed to the momentum mode of the open string, i.e., photon on the D-brane.

Since we have found the vector potential A_a representing the fundamental string, our next task is to obtain the matrix representation X_a of the covariant derivative (11). Taking the basis $\phi_m(n_1, n_2, n_3) \equiv \exp \left(i \sum_{i=0}^3 n_i x_i / \tilde{R}_i \right) e_m$, where $e_m = (0, \dots, 1, \dots, 0)^T$ is a unit vector whose m th component is 1, we obtain

$$\begin{aligned} \nabla_1 \vec{\phi}(n_1, n_2, n_3) &= \frac{n_1}{\tilde{R}_1} \vec{\phi}(n_1, n_2, n_3) \\ &+ \frac{c}{2} e^{-i \frac{t}{\tilde{R}_3}} H_i \vec{\phi}(n_1, n_2, n_3 + 1) + \frac{c}{2} e^{+i \frac{t}{\tilde{R}_3}} H_i \vec{\phi}(n_1, n_2, n_3 - 1), \end{aligned}$$

$$\begin{aligned}\nabla_2 \vec{\phi}(n_1, n_2, n_3) &= \frac{n_2}{\widetilde{R}_2} \vec{\phi}(n_1, n_2, n_3), \\ \nabla_3 \vec{\phi}(n_1, n_2, n_3) &= \frac{1}{\widetilde{R}_3} \left(n_3 + \sum_k a_k H_k \right) \vec{\phi}(n_1, n_2, n_3).\end{aligned}\tag{18}$$

The first terms on the RHS of these equations exist even in the case where photon is absent. They correspond to background D3-brane (or D-particle before T-dualization). Therefore the components which represent the fundamental string (photon) are

$$(X_1)_{(n_1, n_2, n_3), (n_1, n_2, n_3 \pm 1)} = \pi c H_i \exp \left(\mp i \frac{t}{\widetilde{R}_3} \right).\tag{19}$$

The oscillation in time is due to the energy of the photon.

The correspondence between fluctuations of non-diagonal part of X_a and an open string stretched between two D-particles is a fundamental concept in M(atr)ix theory. So far this non-diagonal part has been treated only as a virtual string which contributes to a potential between D-objects. We emphasize that our solution (19) represents not the virtual string but the static real BPS string.

Eq. (19) represents a wrapped string whose both the ends are on the same D-particle i (A in Fig. 1). In this case, the photon is neutral for all the $U(1)$ s, which are the remnants of $U(N)$ gauge symmetry (we are considering the generic case where gauge symmetry is broken to $U(1)^N$). In addition to these N neutral photons, $N^2 - N$ charged particles, which correspond to the non-diagonal part of $U(N)$, also exist. We call them “W-boson”. Every W-boson is charged with respect to two of the N $U(1)$ s. These W-bosons represent a string whose ends are on different D-particles corresponding to the two $U(1)$ s (B in Fig. 1). However this configuration is unstable and cannot be an exact solution of M(atr)ix theory, since the string shrinks due to its tension and the D-particles cannot stand still. To make it stable, we must add another string to pull the D-particles to the opposite direction (C in Fig. 1). In SYM theory, this corresponds to the fact that a single W-boson cannot exist in a compact space (T^3 in this case) because flux has nowhere to go, while two W-bosons whose charges are opposite each other can exist (these W-bosons correspond to two strings of C in Fig. 1).

In general, there exist stable states with arbitrary number of W-bosons. For such states we can prove the equivalence between “the condition of balance of the tension” and “the charge vanishing condition” as follows. If the tensions of the strings attached to each D-particle are balanced, the energy of the system is stationary against small shifts of the D-particles. In SYM theory, this corresponds to the fact that small changes of the Wilson lines keep the energy unchanged. The energy of this system is the sum of the energies E_i of the W-bosons

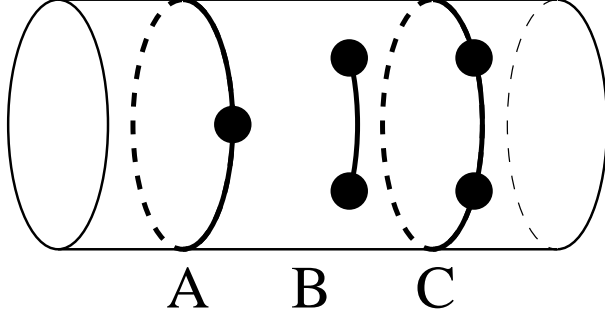


Figure 1: Wrapped string corresponding to a neutral photon (A), unstable configuration corresponding to a single charged W-boson (B) and stable configuration corresponding to two W-bosons whose charges are canceled out (C). The blobs represent D-particles.

(index I labels the W-bosons). They are given by

$$E_I = 4\pi^2 R_3 \left| \sum_k q_{Ik} a_k + n_I \right|, \quad (20)$$

where q_{Ik} and n_I are the charges of W-bosons and the integral numbers labeling the Kaluza-Klein modes, respectively, and index k specifies one of the N $U(1)$ s. The signature of the quantity inside the absolute value on the RHS of (20) corresponds to the winding directions of the string. Because the BPS saturation condition demands all these direction to be the same, we can ignore the absolute value symbol. Hence, the change of the total energy $E_{\text{total}} = \sum_I E_I$ caused by the shift of the Wilson lines is

$$\frac{\partial E_{\text{total}}}{\partial a_k} = 4\pi^2 R_3 \sum_I q_{Ik}. \quad (21)$$

Therefore, the stationarity of the total energy means that the sum of the charges of W-bosons vanishes.

Taking these points into account, we can easily construct exact solutions of the equation of motion $\ddot{X}_a + [X_b, [X_b, X_a]]/(2\pi)^2 = 0$ as a sum of the oscillation modes representing each W-boson:

$$\begin{aligned} X_1 &= X_1^{(0)} + \sum_{I=1}^M X_1^{(I)}, \\ (X_2)_{(n_1, n_2, n_3), (n_1, n_2, n_3)} &= 2\pi R_2 n_2, \\ (X_3)_{(n_1, n_2, n_3), (n_1, n_2, n_3)} &= 2\pi R_3 \left(n_3 + \sum_k a_k H_k \right), \end{aligned} \quad (22)$$

where

$$\begin{aligned}
(X_1^{(0)})_{(n_1, n_2, n_3), (n_1, n_2, n_3)} &= 2\pi R_1 n_1, \\
(X_1^{(I)})_{(n_1, n_2, n_3), (n_1, n_2, n_3)} &= \pi c \left(\exp(+i(a_{i_{I+1}} - a_{i_I})t) E_{i_I i_{I+1}} + \text{h.c.} \right) \quad \text{for } 1 \leq I \leq M-1, \\
(X_1^{(M)})_{(n_1, n_2, n_3), (n_1+1, n_2, n_3)} &= \pi c \exp(+i(a_{i_1} + 1 - a_{i_M})t) E_{i_M i_1}, \\
(X_1^{(M)})_{(n_1+1, n_2, n_3), (n_1, n_2, n_3)} &= \pi c \exp(-i(a_{i_1} + 1 - a_{i_M})t) E_{i_1 i_M}.
\end{aligned} \tag{23}$$

In eqs. (22) and (23), i_I and i_{I+1} are the numbers which satisfy $q_{I i_I} = +1$ and $q_{I i_{I+1}} = -1$, M is the total number of the W-bosons, and we have assumed that $0 \leq a_{i_1} < a_{i_2} < \dots < a_{i_M} < 1$.

5 Conclusion

In this paper we have obtained a solution of M(atrrix) theory corresponding to fundamental string. It is given as an oscillation mode of non-diagonal part of matrices X_a , eq. (19), and represents a static BPS string.

There are a number of questions to be clarified. For example, we do not know how to reproduce the following three in M(atrrix) theory:

- Infinite tower of massive modes of string.
- Non-wrapping closed string, in particular, graviton.
- Winding BPS states of closed string.

We shall comment on the first and the last objects.

Concerning the infinite tower of massive modes of string, we should comment on [8, 9, 10]. Although they reproduce the massive modes of string, the massive modes are the ones of the string in the S-dualized description (recall the explanation in Sec. 1). What we would like to reproduce here is the massive modes in non-S-dualized description, which correspond to the excitation modes of D-objects in the S-dualized description (however, they are not discussed in [8, 9, 10]).

Next we shall comment on the winding closed string. If a solution representing the winding closed string (B in Fig. 2) exists, it must satisfy eq. (15) like the open string states we have discussed (A in Fig. 2). However, every element X_{mn} corresponds to the string stretched between D-particles. So within our framework there is no way to describe winding closed strings which are not connected to D-particles. Two types of states (A and B in Fig. 2) have

the same energy, and transition between them must exist to obey unitarity, so that the winding closed string state should also exist in the theory.

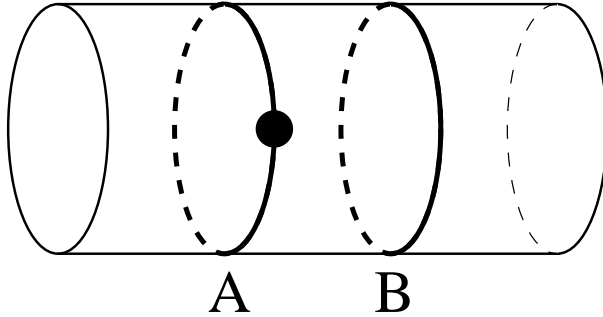


Figure 2: An open string (A) pinned to a D-particle (blob) and a winding closed string (B).

I would like to thank H. Hata and T. Kugo for valuable discussions and careful reading of the manuscript.

Note added: After completing this paper, I became aware of ref. [13], which gives a similar argument as mine. I would like to thank Pei-Ming Ho for informing me of their work.

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